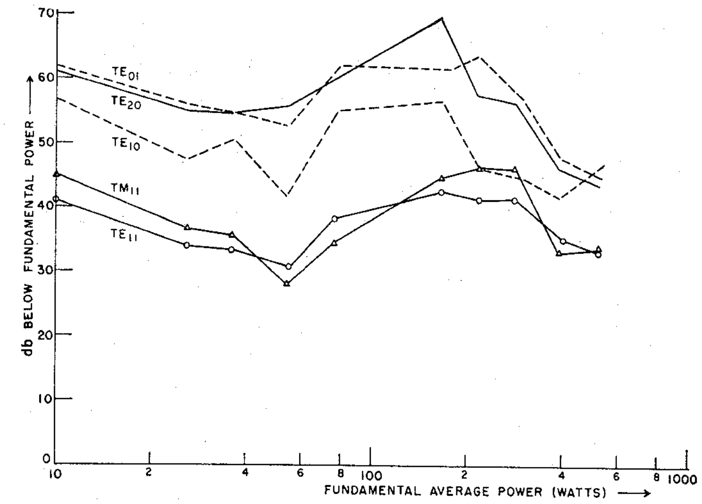

Fig. 16—Second harmonic average power content in the TE_{10}, TE_{20}, TE_{01}, TE_{11}, and TM_{11} modes.

ducer used with the 5586 magnetron. Evidently the concentration of power in the TE_{11}, TM_{11}, and TE_{10} modes is due to the symmetry of this transducer.

The couplers used to obtain the curves of Fig. 16 were adapted for use in a pressurized system. However, at the power levels involved (500 kw) arcing was not a problem and pressurization was not used. These measurements were made by Dr. Pietro Lombardini [7].

Conclusions

Experimental models of the mode couplers indicate that mode selectivities of the order of 30 db or greater over bandwidths of 200 mc or greater are obtainable in simple two-hole couplers. Degenerate modes pose a special problem and may result in substantially lower selectivities. As many as four modes can be simultaneously rejected while coupling strongly to a fifth. The application of these couplers seems limited to systems in which the number of propagating modes is small, but within this limitation they provide a quick and covenient power measurement technique.

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Measurement of Harmonic Power Generated by Microwave Transmitters*

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Summary—**A measurement technique is described that can be used to determine quantitatively the power levels of the higher order modes propagating in a straight, lossless, rectangular waveguide. The technique employs a number of small calibrated electric probes which are fixed on the broad and narrow walls of the waveguide measurement section to sample the electric fields within. The method used to calibrate these probes is briefly discussed, and information on accuracy and limitations of the probe technique is presented. Some measurement results on the power levels in the modes of the second and third harmonic frequencies in the outputs of high power *S*-band magnetrons and klystrons are presented.**

The multiple-probe technique has reduced the time required to take measurements at a given frequency to about one-half hour. An automatic computer has been programmed to perform all of the required mathematical operations and has reduced the computation time to less than one-half hour for each measurement frequency.

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Introduction

In recent years several workers have advanced methods of measurement of the harmonic frequency power in the output of microwave tubes. Interest in these measurements has been stimulated to a large extent by the acute problems of radio interference between microwave systems which can result from unwanted radiation of harmonic frequency energy. However, measurement of this power is complicated by the fact that harmonics in waveguide lines may propagate in many different modes which are convertible one to another by the presence of obstacles or ports. This paper presents one of the possible techniques for measurement of the powers in higher order modes at harmonic frequencies in waveguides. To accomplish this measurement, a number of small, fixed electric probes are located on the walls of a waveguide measurement section to sample the electric field within. From each of these probes a measurement of relative phase angle and absolute power output is made with a frequency selective microwave receiver tuned to the particular harmonic (or spurious nonharmonic) frequency of interest. It is necessary, of course, that each probe and its associated cabling be calibrated so that a measured power output level will correspond to a calculable value of electric field within the waveguide. From this measurement of probe output power and relative phase angle, one can compute the power level in each of the modes which are above cutoff at the measurement frequency.

The mathematical analysis underlying the present technique is identical with that presented by Forrer and Tomiyasu,[1,2] although the laboratory procedures are different. For example, they used a sliding electric probe on the broad wall of a section of waveguide which they positioned in both the transverse and longitudinal dimensions to sample the electric fields within. The sliding probe, while electrically simple, is cumbersome to employ, and avoidance of voltage breakdown under high power was difficult. To avoid arcing, they found it necessary, in fact, to turn off the high power source each time the sliding probe was moved to a new position. This procedure not only was time consuming, but it also gave rise to possible inaccuracies, because it was not certain that the source when turned on again at each new probe position would be operating as it was during the previous position.

In the technique used by Lewis,[3] a direct measurement of the power in higher order modes is possible by the use of a set of calibrated mode selective couplers. With this method, Lewis was able to measure the power of five higher order modes at the second harmonic of an *S*-band signal. It probably would be difficult, however, to design a set of mode couplers which could be used for measurements in which dozens of higher order modes are propagating. In the use of the multiple probe technique, on the other hand, it has been possible to measure the power in eleven modes at a third harmonic frequency of an *S*-band signal with good accuracy. Measurements at a frequency in which twenty-five modes propagate can be made with reduced accuracy. In addition, it is possible to measure all of the quantities necessary to determine the modal powers in only a few minutes at any one frequency as contrasted with several hours required in the sliding probe method.

[1] M. P. Forrer and K. Tomiyasu, "Effects and measurement of harmonics in high power waveguide systems," 1957 IRE National Convention Record, pt. 1, pp. 263–269.

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[3] D. J. Lewis, "Mode couplers and multimode measurement techniques," this issue, p. 110.

In the following paragraphs, a brief outline is presented of the measurement method used. The technique employed in calibration of the probes is discussed, and the results of some high power tests are presented with an estimate of the measurement accuracy.

Outline of Measurement Technique

The electric probe method is based on the fact that one can determine the maximum number of modes in which a signal might propagate in a waveguide at each frequency of interest. A chart of the cutoff frequencies for two waveguide sizes is given in Fig. 1. For example,

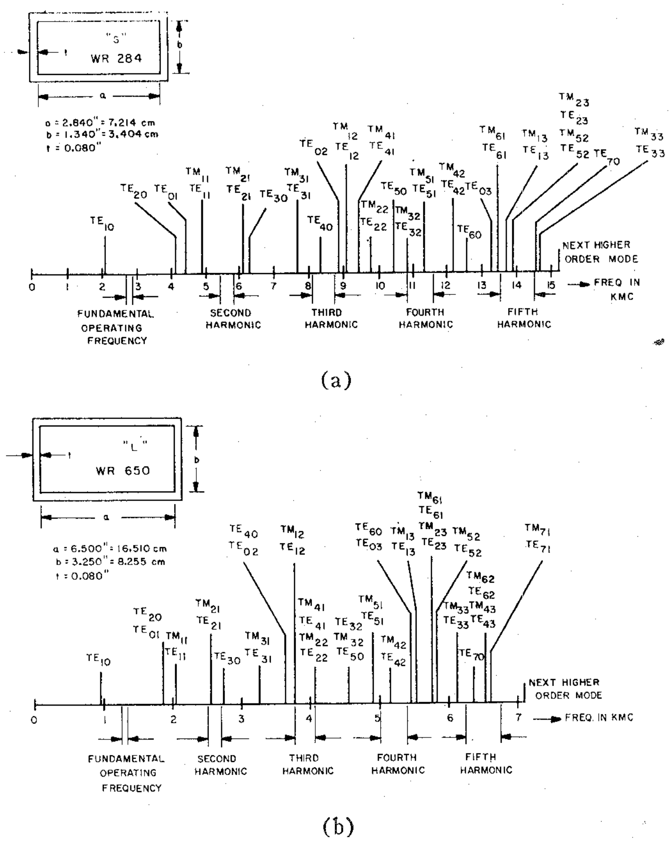

Fig. 1—Chart of mode cutoff frequencies for (a) *S*-band and (b) *L*-band waveguides.

in WR-284 waveguide, a signal at 5 kmc could propagate in the TE_{10}, TE_{20}, TE_{01}, TM_{11}, and TE_{11} modes but no others. With this knowledge, one can make an analysis. The analysis requires that depending on the modal complexity, measurements must be made at a number of prescribed locations on the waveguide walls. An illustration of a WR-650 waveguide section with its probes in position is shown in Fig. 2. In general, the minimum number of probes required is slightly more than the number of modes which can propagate at a given frequency. For example, the number of probes required at each broad-wall cross section is equal to the highest m-index to occur for a propagating mode. Similarly, the number of probes required at a cross section on the narrow wall is equal to the highest n-index to occur for a propagating mode. Thus, to measure the TE_{71} and TM_{43}

modes, one needs at least seven equally spaced broad-wall probes and three equally spaced narrow wall probes. The number of measurement cross sections required is determined by the following relationships:

Number of broad-wall cross sections $=1+n_{\max}(1)$
Number of narrow wall cross sections $=1+m_{\max}(1)$

where $n_{\max}(1)$ is the maximum n-index to occur in a propagating mode whose m-index is unity and $m_{\max}(1)$ is the maximum m-index to occur in a propagating mode whose n-index is unity. Thus, if the TE_{71} mode were propagating, probe measurements would need to be made at eight narrow wall cross sections. Also, if the TE_{13} mode were propagating, probe measurements would need to be made at four broad-wall cross sections.

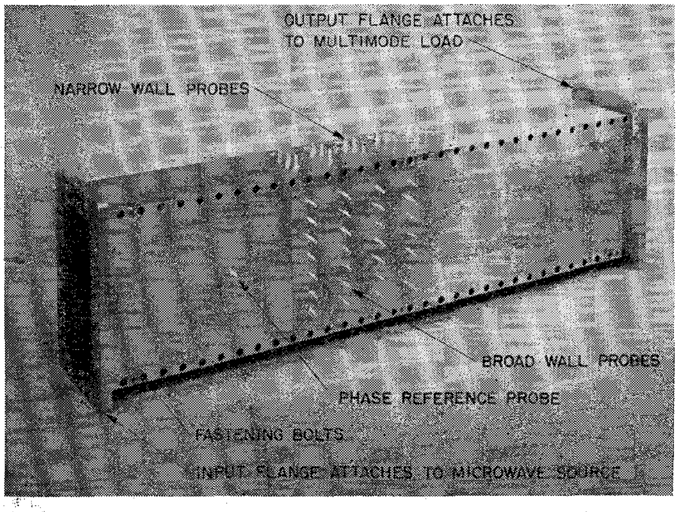

Fig. 2—Illustration of a probe section fabricated in WR-650 waveguide.

The probes at each cross section protrude only a few mils into the waveguide for minimum perturbation of the fields; hence, one can assume that the electric field amplitude of each mode will vary as a sine function over the cross-sectional dimension of the waveguide. Under this assumption, one can use the principle of numerical Fourier analysis to determine the values of the electric field phasors at each probe position. If these phasors are separated into their real and imaginary parts and referred to one cross section, one can write a set of linear equations, the solution of which gives the desired values of electric field intensities for each of the modes of interest. The power in the modes may then be computed readily by integrating Poynting's vector over the waveguide cross section.

This mathematical computation is quite time consuming even for just a few modes. To reduce the computation time, a program has been written for the IBM 650 and 704 computers to perform all of the required steps once the experimental data is obtained. It has been found possible to compute the power in 44 modes at the fifth harmonic of an L-band source in less than twenty minutes, including punching the input data cards and the solution print-out.

A block diagram of the apparatus used to measure the absolute power and relative phase angle at each probe is shown in Fig. 3. Coaxial switches are used to connect the probes to the measurement set.

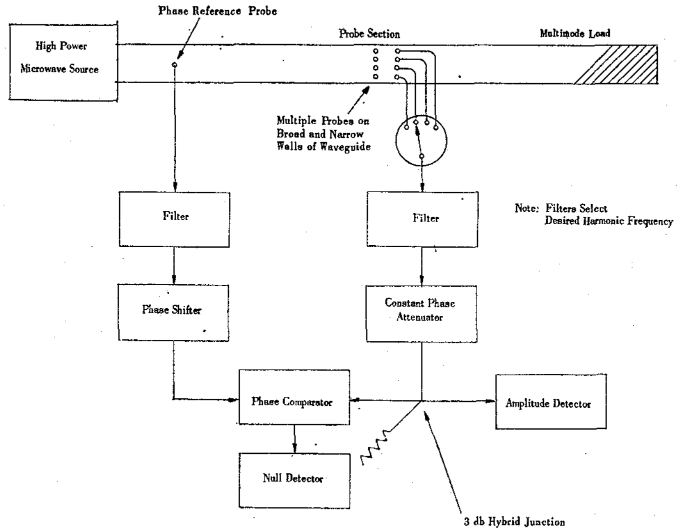

Fig. 3—Simplified block diagram of phase and amplitude measurement equipment.

Probe Calibration Procedure

The function of the probes is to sample the electric field at various prescribed locations on the waveguide wall. The magnitudes and relative phases of the electric fields at the probes are quantities from which the power levels of the several propagating modes of interest may be computed. However, since the magnitude of the electric field at a probe is determined by a measurement of the power output at the end of its connecting cable, it is necessary to find the relationship between the electric field E_i at the ith probe and this power output, P_i. The relationship may be expressed by the equation

$$| E_i |^2 = \frac{P_i}{C_i} \tag{1}$$

where C_i is the probe coupling coefficient which is unique to the ith probe.

To evaluate C_i experimentally, a measurement of P_i is made for each probe with a known value of $|E_i|$ due to a single mode propagating in the probe section. In order to establish a known value of $|E_i|$, a single mode at the desired harmonic frequency is launched into the probe section and, to insure only forward propagating modes, a suitable multimode load is used. To calibrate the broad-wall probes, the TE_{10} mode is used and is launched by means of a very gradual waveguide transition. The power in this mode flowing in the z-direction is given by

$$W_z^{TE_{10}} = G^{TE_{10}} | E_{\max}^{TE_{10}} |^2 \tag{2}$$

where $|E_{\max}^{TE_{10}}|$ is the maximum amplitude of the electric field of the TE_{10} mode and $G^{TE_{10}}$ is a calculable quantity defined in the following equation:

$$G^{TE_{10}} = \frac{a^3 b \epsilon_0 f_c^2}{2c} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (3)$$

where

a = broad-wall dimension of waveguide,
b = narrow wall dimension of waveguide,
ϵ_0 = free space dielectric constant,
c = velocity of light,
f_c = cutoff frequency for TE_{10} mode,
f = operating frequency.

In the mathematical analysis, it is assumed that the broad-wall probes at each cross section are spaced at p equal intervals across the waveguide walls. Therefore, the power output of the ith probe due to the calibrating mode is given by

$$P_i^{TE_{10}} = C_i \left| E_{max}^{TE_{10}} \right|^2 \sin^2 \frac{i\pi}{p}. \quad (4)$$

Solving (2) and (4) for $|E_{max}^{TE_{10}}|$ and equating yields

$$C_i = \frac{G^{TE_{10}}}{L_i \sin^2 \left(\frac{i\pi}{p}\right)} \quad (5)$$

where $L_i = W_z^{TE_{10}}/P_i^{TE_{10}}$ is the insertion power ratio of the ith probe. This ratio is determined for each broad-wall probe by using an RF substitution technique.

The value of C_i, given by (5), may now be inserted into (1) to get an expression for the electric field $|E_i|$ at a probe:

$$|E_i|^2 = \left[\frac{L_i \sin^2 \left(\frac{i\pi}{p}\right)}{G^{TE_{10}}} \right] P_i. \quad (6)$$

Eq. (6) is used for the measurement of the electric fields on the broad wall, and a similar equation is used for the narrow wall probes. In the latter case, a single TE_{01} mode is launched into the probe section with suitable termination to provide the desired calibration information.

The mathematical analysis requires that measurements be made of the probe power outputs as well as the relative phase angles from one probe to another. Therefore, it is necessary, as part of the calibration procedure, to measure the relative phase angles of the probes and their attached cables and switches when a single TE_{10} or TE_{01} mode is launched into the probe section. During fabrication of the probe section, every attempt is made to make the length of the cables attached to each probe the same. However, it is not practical to make the electrical lengths of these equal to $n\pi$ with tolerances closer than about ± 10 degrees. Hence, during the phase calibration procedure, the exact electrical length of the lines to each probe (to within ± 0.5 degree) is measured and recorded.

Measurement Results

The harmonic output powers of two types of S-band tubes were measured. These types were a 4.7-megawatt S-band magnetron and a 1.3-megawatt S-band klystron. In these tests the tubes were operated at the electrode voltage ratings and drive conditions recommended by their manufacturers. If the tubes are measured when operating at lower or higher values than those recommended, it has been observed that the harmonic content can change by a large amount. Table I is a tabulation of the harmonic power output of an S-band klystron and magnetron. In the tubes tested, no directed efforts were made to reduce the harmonic frequency outputs of the tubes.

TABLE I
Harmonic Power Output

	S-Band Magnetron	S-Band Klystron
Fundamental TE_{10}	4.7 Megawatts (0-db reference)	1.3 Megawatts (0-db reference)
2nd Harmonic		
TE_{10}	105 watts	23 watts
TE_{20}	—	6
TE_{01}	—	6
TE_{11}	—	13
TM_{11}	25	60
Total 2nd harmonic power	130 watts (−45.5 db)	108 watts (−41 db)
3rd Harmonic		
TE_{10}	4356 watts	4 watts
TE_{20}	24	—
TE_{30}	6036	—
TE_{40}	22	—
TE_{01}	1	5
TE_{11}	7	1
TE_{21}	2	—
TE_{31}	10	1
TM_{11}	6	1
TM_{21}	6	—
TM_{31}	15	—
Total 3rd harmonic power	10,485 watts (−26.5 db)	12 watts (−50.4 db)

Limitations and Accuracy

While the multiple probe technique allows one to determine the power levels of a large number of modes in a waveguide, it does have some limitations. As the modal order increases, so does the order of the determinant that must be solved; therefore, the number of modes one can measure is limited both by the storage capacity of the computer available and by the large errors that can occur in the solution of high-order determinants even for small errors in the probe readings which supply the experimental data for the determinants. The order of the determinants to be solved is related to the mode indices by the following:

Order of determinant for broad wall $= 2[1 + n_{max}(1)]$

Order of determinant for narrow wall $= 2[1 + m_{max}(1)]$.

Or, the order of determinants is twice as great as the number of cross sections required to evaluate the modes at any frequency. Thus, to evaluate the modes which may propagate at 8500 mc in WR-284 waveguide, determinants of order as high as eight must be solved for the narrow wall probes and determinants of order as high as four must be solved for the broad-wall probes. If one wished to measure the power in the modes at 7000 mc in WR-650 waveguide, however, determinants of order as high as sixteen would need to be solved, and it is with determinants of this size that large errors can occur in the computed modal power for slight inaccuracies in measurement of probe outputs. It is not likely that this is a basic limitation of the electric probe technique, but rather of the particular computer program written to carry out the mathematical operations. The inclusion of redundant experimental data having finite accuracy into a computer program designed to accommodate it might make possible accurate measurement of a large number of higher order modes.

It is of importance to consider the accuracy, repeatability, and resolution of the apparatus needed to make the measurements with the probe technique. A determination of the accuracy of the results has been made by launching a TE_{20} mode with good single mode purity and known power level into the probe measurement section and its multimode load. The outputs from each probe were measured and the data were reduced with an automatic computer. The measured power input was 201 milliwatts, while the power as indicated in the computer results was 196 milliwatts in the TE_{20} mode, 1.5 milliwatts in the TE_{10} mode, and a few microwatts in other propagating modes.

Experience with measurements of the harmonic output of a number of high power tubes has shown that, if the environmental conditions are unchanged, the power levels measured at each probe are repeatable within ± 1 db, while the phase angles recorded at each probe are repeatable within $\pm 2°$. This repeatability is possible even though the high power tubes are operated under pulsed conditions. An analysis was made of the change in reported modal power in the computer results if the data input to the computer were altered in random manners with changes in both probe powers and probe phase angles. As a result of a series of tests with random variations in the data, the probable modal power levels reported for the magnetron and klystron are within ± 10 per cent of their true values. Under the ideal conditions obtained during calibration operations, the measurement apparatus itself is capable of resolving changes of ± 0.1 db and ± 0.5 degree of phase angle.

Conclusions

A method of measurement of the harmonic power in rectangular waveguide has been presented which has some advantages over other techniques for higher order mode measurement. The use of multiple probes on the waveguide walls, as contrasted with the sliding probe of Forrer and Tomiyasu, has proved to be mechanically superior and has reduced the time required to obtain the experimental data to less than a half hour at each frequency. It is probable that more modes could be measured with the multiple probe technique than with the method of Lewis. The procedure used to calibrate the probes with known signals was discussed and comparison was made of the power levels in the harmonics of high power *S*-band magnetrons and klystrons. Some of the limitations and an estimate of the accuracy are included.

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